

Large Scale Nuclear Motion

On the Way to Super Heavies

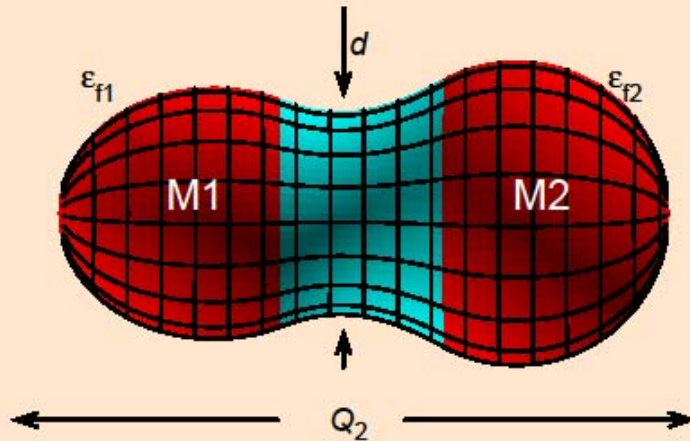
Hans Feldmeier, GSI

Outline

- Collective variables
- Energy landscape $V(\mathbf{q})$, mass tensor $M(\mathbf{q})$
- Dissipation fluctuation $\gamma(\mathbf{q})$, $D(\mathbf{q})$
- How to make heavy system fuse
- Shell effects
- Quantum correlations (beyond mean field)
- Microscopic consistent input to $V(\mathbf{q}), M(\mathbf{q}), \gamma(\mathbf{q}), D(\mathbf{q})$

Collective Variables $q = \{q_1, q_2, \dots\}$

Five Essential Fission Shape Coordinates



41	$Q_2 \sim$ Elongation (fission direction)
⊗	
20	$\alpha_g \sim (M1-M2)/(M1+M2)$ Mass asymmetry
⊗	
15	$\epsilon_{f1} \sim$ Left fragment deformation
⊗	
15	$\epsilon_{f2} \sim$ Right fragment deformation
⊗	
15	$d \sim$ Neck

⇒ 2 767 500 grid points – 156 615 unphysical points

⇒ **2 610 885 physical grid points**

Example with axial symmetry:

q_1 elongation

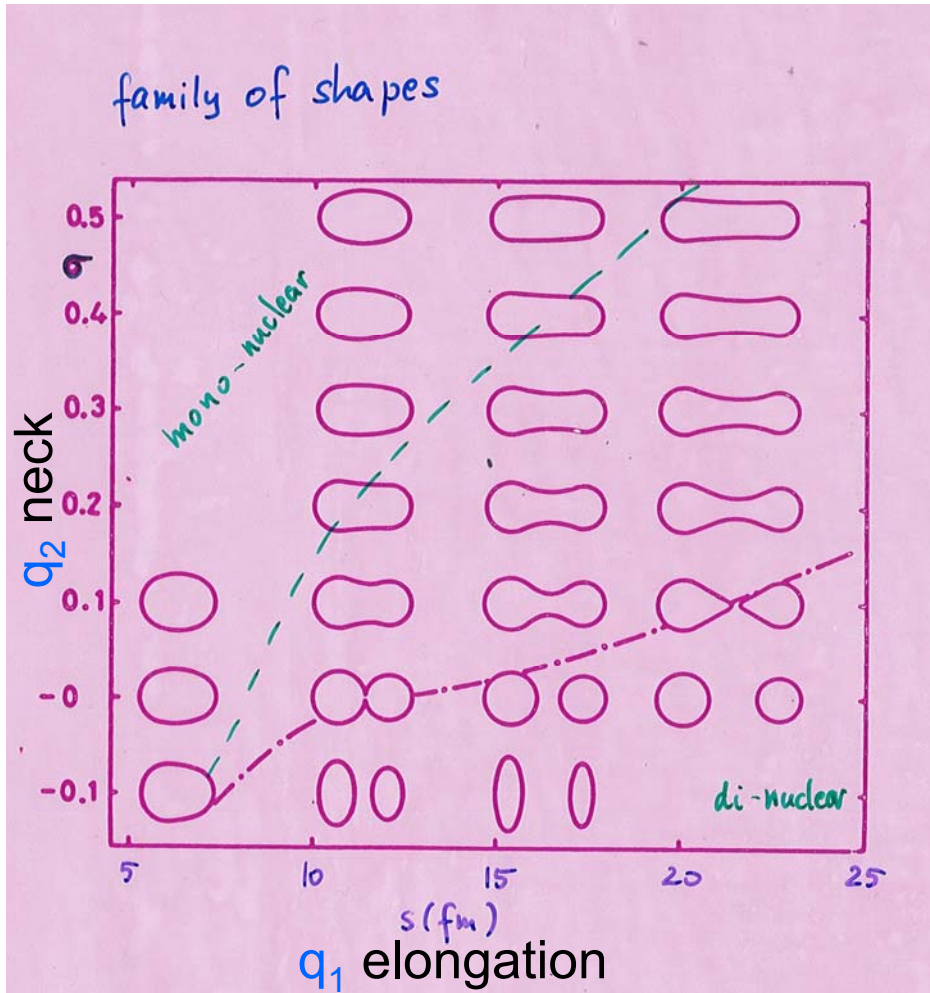
q_2 neck

q_3 mass asymmetry

q_4 deformation left

q_5 deformation right

Collective Variables $q = \{q_1, q_2, \dots\}$

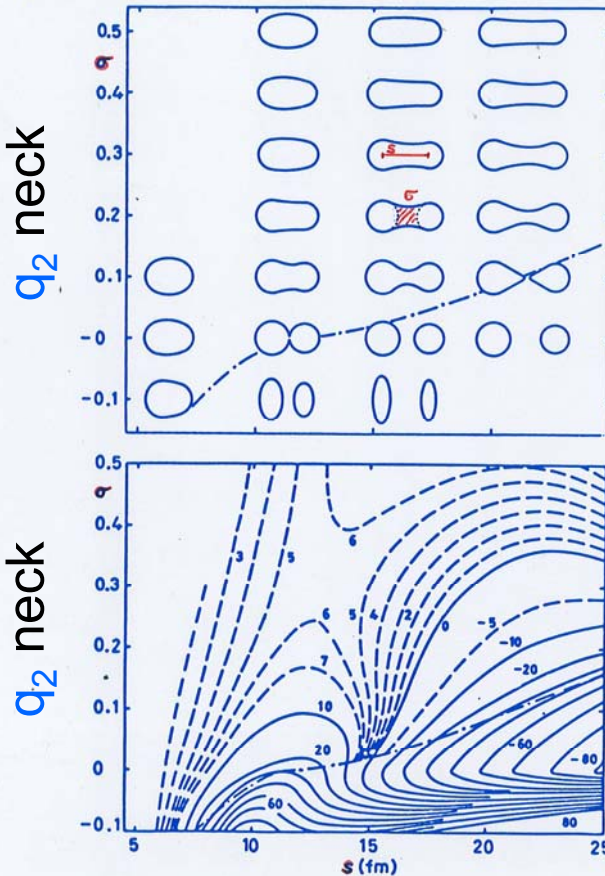


- Family of shapes characterized by coll. variables
- like quadrupole, octupole etc. moments

Energy Potential Landscape $V(q)$

Macroscopic variables

q_1 elongation



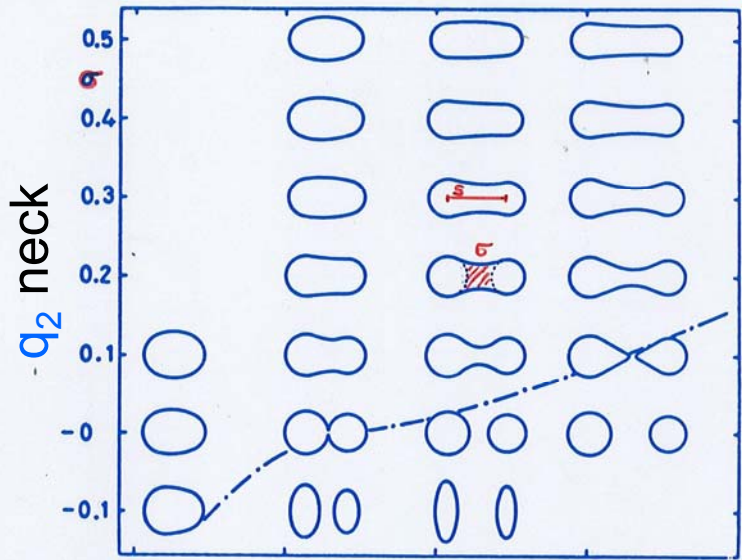
Potential: folding potential (Krappe, Nix, Sierk)

$$V(s, \sigma, \alpha) = \underbrace{\frac{1}{2} \rho_c^2 \int d^3r d^3r' \frac{1}{|\vec{r} - \vec{r}'|}}_{\text{Coulomb } b} + \underbrace{\frac{1}{2} \rho_s \int d^3r d^3r' \left(\frac{1}{a} - \frac{2}{|\vec{r} - \vec{r}'|} \right) e^{-\frac{|\vec{r} - \vec{r}'|}{a}}}_{\text{nuclear}}$$



- Lowest possible energy of quantum many-body system under constraints $\{q_1, q_2, \dots\}$
- $V(q)$ includes all correlations: shell, pairing, vibrations etc.

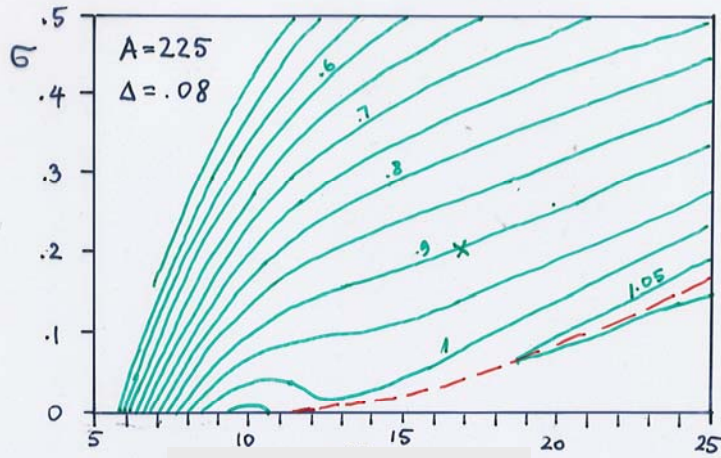
Mass Tensor



Inertial mass tensor $M(q)$

Collective kinetic energy

$$T_{\text{coll}} = \frac{1}{2} \sum_{ij} \dot{q}_i M_{ij}(q) \dot{q}_j$$



q_1 elongation

Werner-Wheeler mass m_{11}/μ

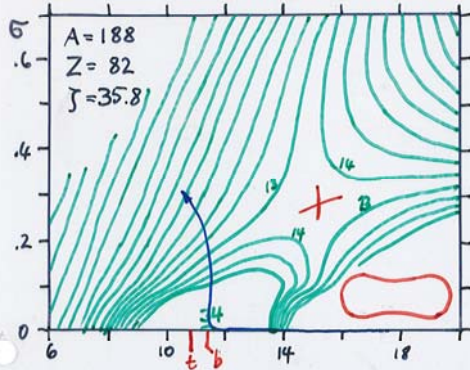
--- scission line

Fissility and Fission Barrier

Fission saddle and saddle-point shapes for increasing fissility

86

▶ Capture takes place if system gets behind fission saddle ◀



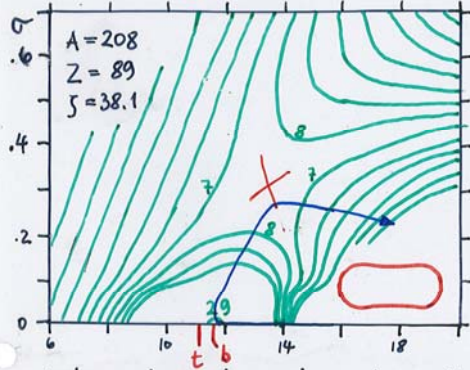
fissility parameter

$$\zeta \equiv \frac{Z^2}{A}$$

$\frac{\text{Coulomb force}}{\text{nuclear force}}$

trajectories:

$\longrightarrow E_{cm} = V_b$

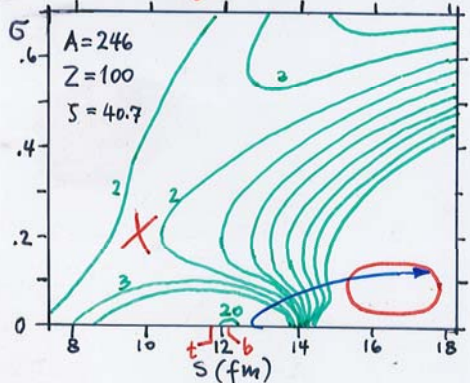


■ symmetric systems ■

$$A_1 = A_2 = A/2$$

$$Z_1 = Z_2 = Z/2$$

■ head on collisions ■

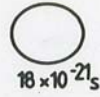
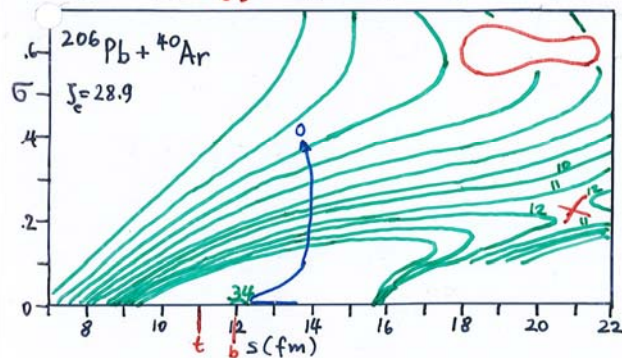
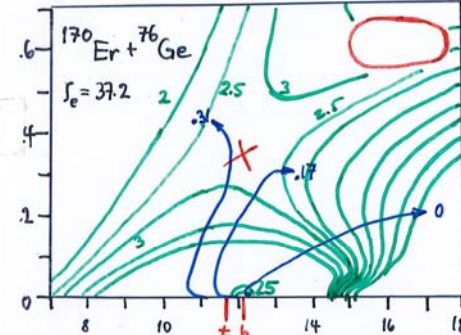
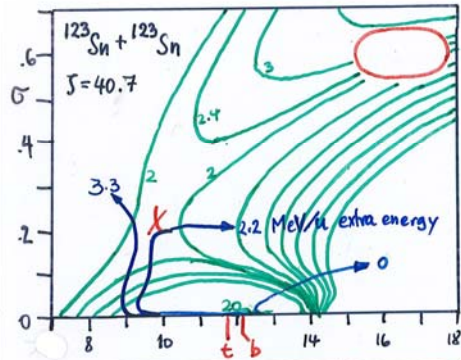


Head-on collisions
Symmetric systems

- Fissility $\zeta = Z^2/A$: measure for ratio Coulomb/Surface energy
- Fission barrier (X) prevents spontaneous fission
- Excited compound nucleus can fluctuate across fission barrier (X)
- Below $\zeta \approx 38$ symmetric system will fuse automatically from top of Coulomb barrier
- For $\zeta > 38$ more beam energy will drive system behind saddle, but compound nucleus highly excited, will fluctuate across low fission barrier (X)

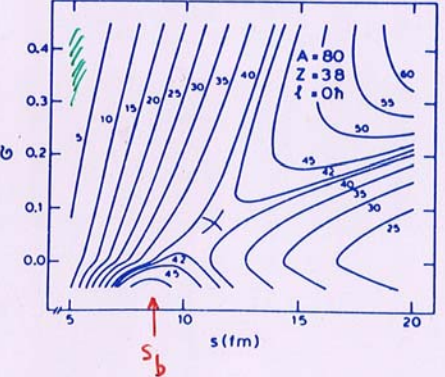
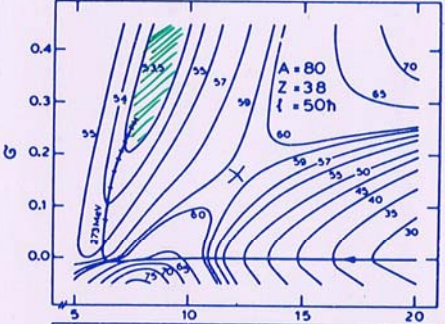
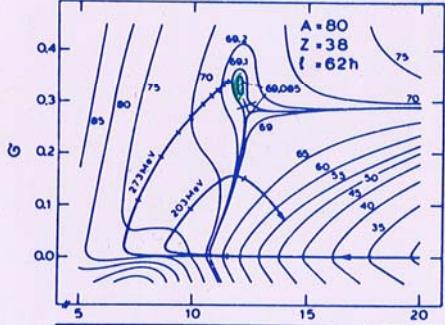
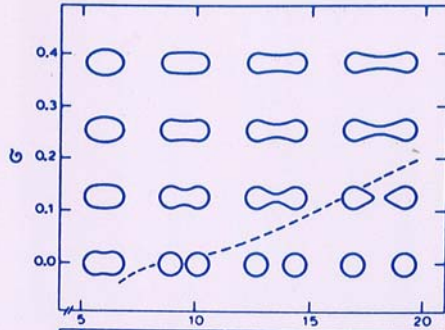
Why Light on Heavy ?

The extra push



- Drive system behind the conditional saddle (X) for given mass asymmetry
- to get finally behind unconditional saddle
- But don't heat up the compound nucleus too much, otherwise it fluctuates out again across the low unconditional saddle

Finite Impact Parameter b (not head-on)



- Larger b (or ℓ) gives more cross section
- $\sigma = \pi b^2$
- For $\ell > \ell_{crit}$ pocket for rotating compound nucleus disappears
- But don't heat up the compound nucleus too much, otherwise it fluctuates out again across the low saddle

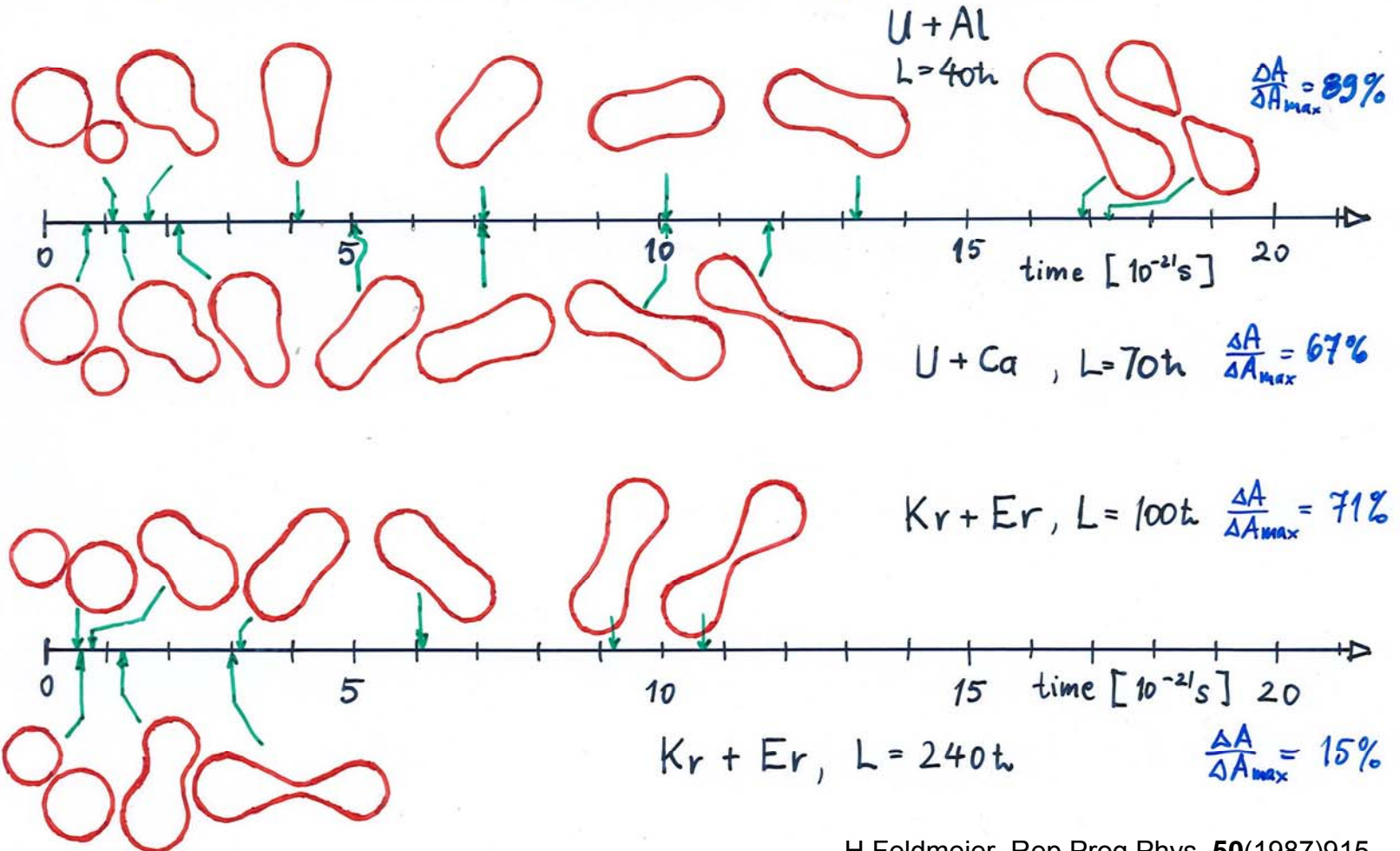
Typical Times

quasi fission



deep inelastic

□ Dynamical evolution of the mass (shape) asymmetry □



Model Ingredients

Conservative part

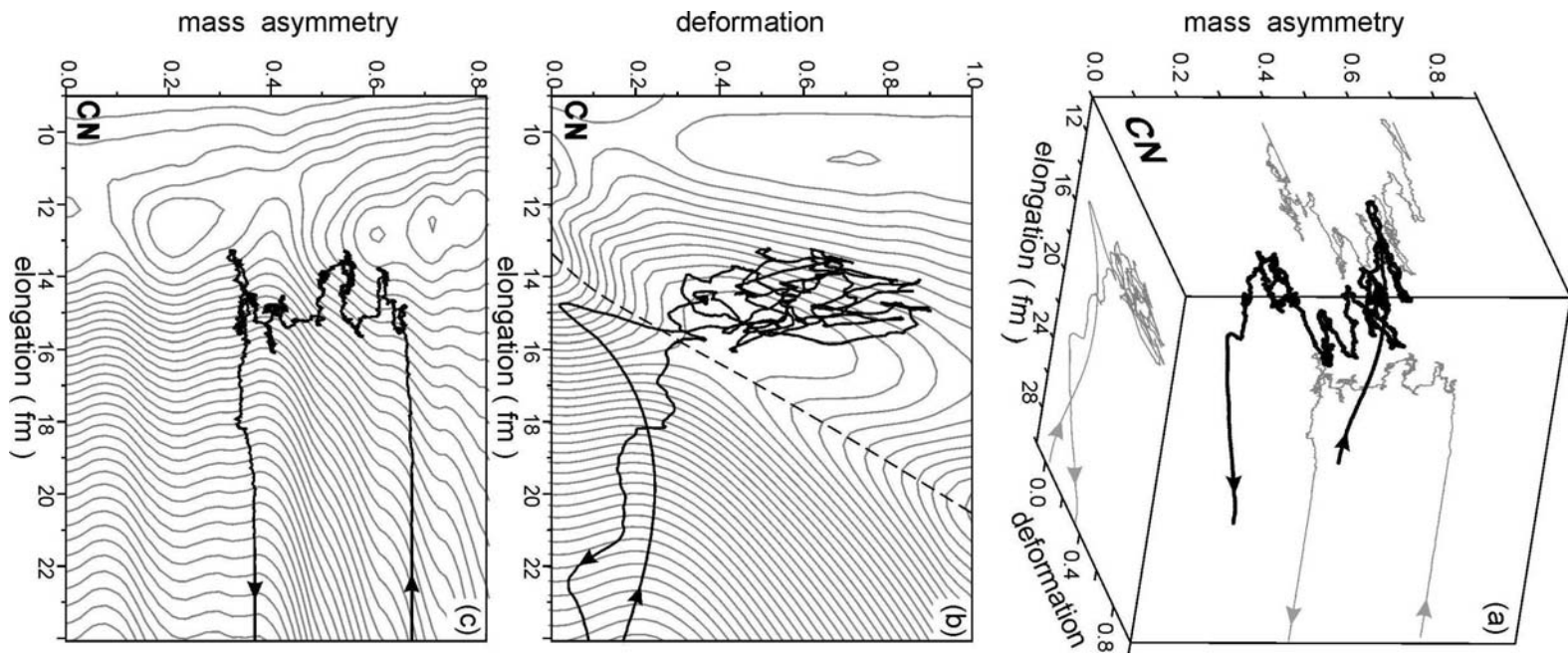
- $q = \{q_1, q_2, \dots\}$ collective variables
- $V(q)$ adiabatic energy (landscape)
- $\dot{q}_i = dq_i/dt$ velocities
- $p_i = \sum_j M_{ij}(q) \dot{q}_j$ collective momenta, $M_{ij}(q)$ mass tensor

Dissipative forces

- $X_i(t) = A_i(q, p) + \delta X_i(t)$ irregular force between intrinsic and coll. variables
- $A_i(q, p) = \langle X_i(t) \rangle \approx \sum_j \gamma_{ij}(q) p_j$ friction force, $\gamma_{ij}(q)$ friction tensor
- $\langle \delta X_i(t) \delta X_j(s) \rangle = 2 D_{ij}(q) \delta(t-s)$, diffusion tensor $D_{ij}(q)$

Langevin Equation with Fluctuating Force

- $dq_i/dt = \Sigma_j M(q)^{-1}_{ij} p_j$
- $dp_i/dt = - dV(q) /dq_i + \Sigma_j \gamma_{ij}(q) p_j + \delta X_i(t)$

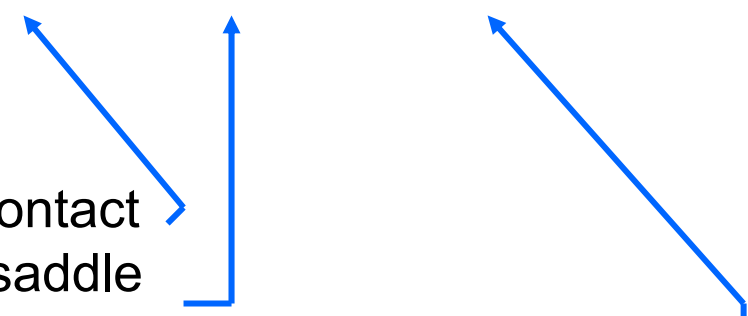


$^{48}\text{Ca} + ^{248}\text{Cm}$ @ 210 AMeV
 fluctating path \rightarrow quasi fission

The 3 Steps to Evaporation Residue (SHE)

Zagrebaev, Greiner, PRC78(2008)034610

$$\sigma_{\text{ER}}^{xn}(E) = \frac{\pi}{k^2} \sum_{l=0}^{\infty} (2l + 1) P_{\text{cont}}(E, l) P_{\text{CN}}(E^*, l) P_{xn}(E^*, l).$$

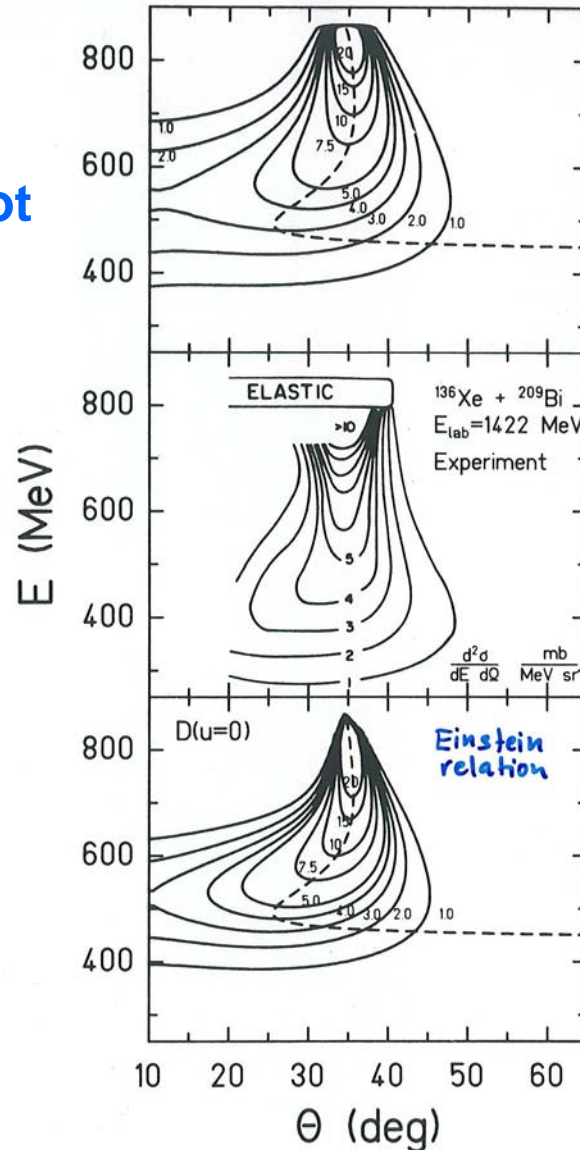
1. Get the nuclei into intimate contact
 2. Get them behind the fission saddle
 3. Hope that they cool down fast enough by neutron evaporation before they fission
- 

Unified microscopic model desirable to go from 1. to 2. to 3. smoothly

Liouville Fokker Planck Equation

Fluctuations in Relative Momentum

Wilczynski plot



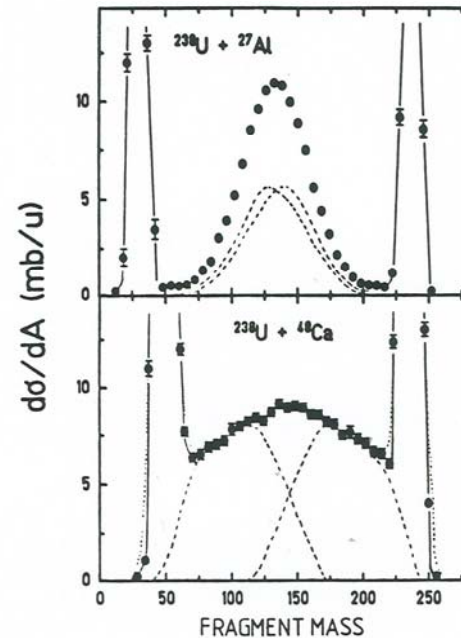
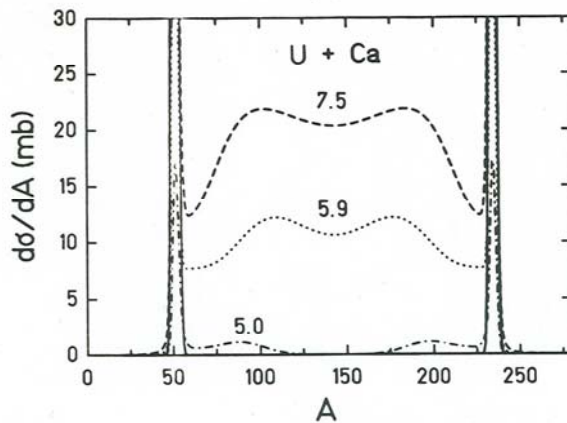
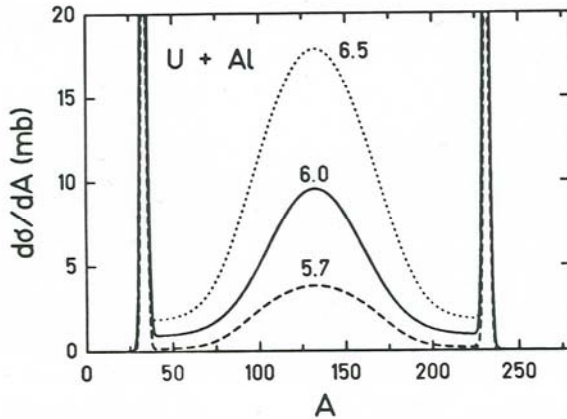
Double-differential cross section for $^{136}\text{Xe} + ^{209}\text{Bi}$. Upper part: result from the particle exchange model including the nonequilibrium velocity part. Centre part: experimental cross section. Lower part: result of a calculation where the Einstein relation has been enforced by setting $\Delta \vec{u} = 0$ at all times in the expression for the diffusion coefficient.



$$E_{\text{lab}} = 1422 \text{ MeV}$$

Drift and Fluctuations in Mass Number

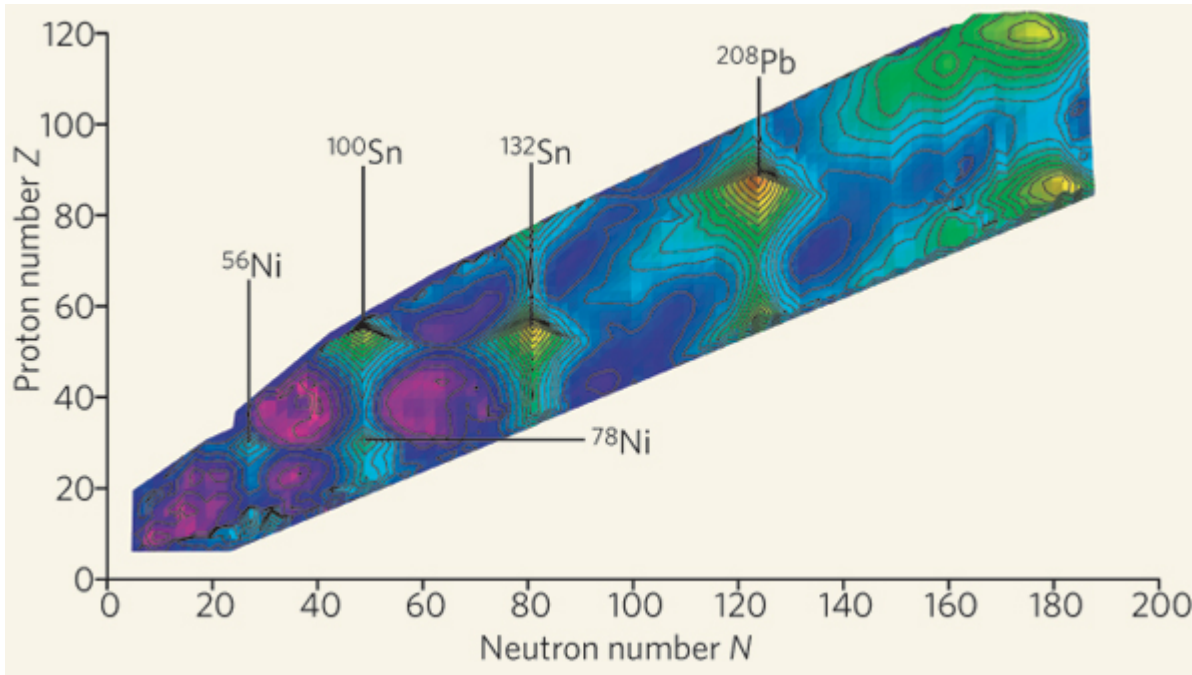
Diffusion plot



quasi fission

deeply inelastic with
large mass drift

Shell Effects



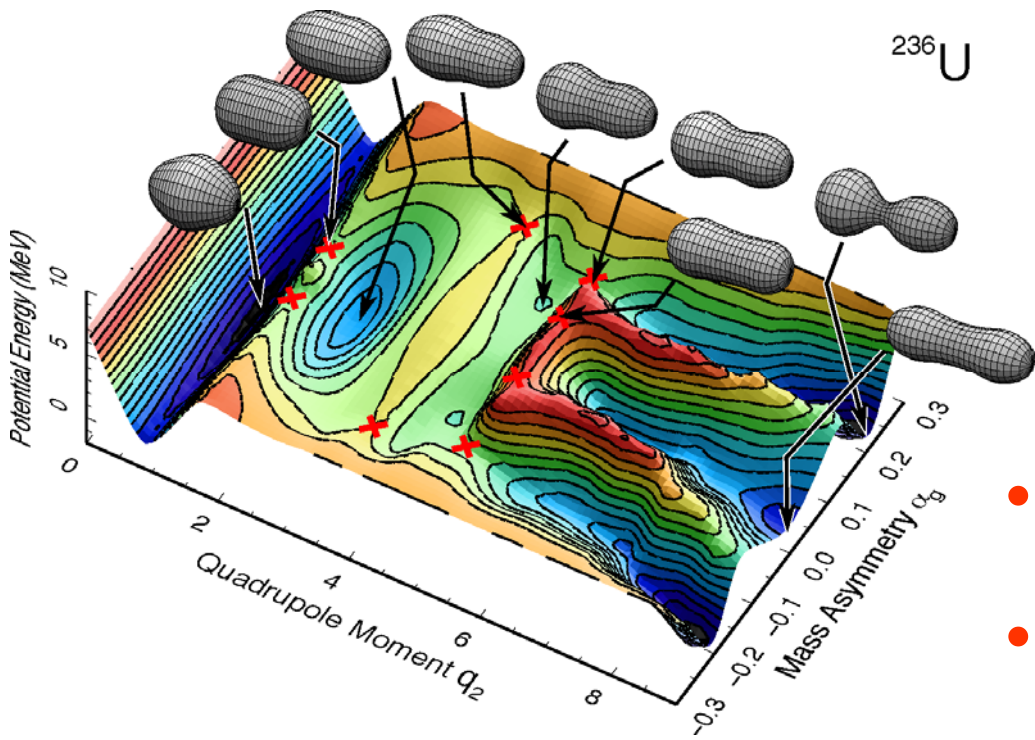
Extra binding due to shell effects up to 10 MeV

Total binding from macroscopic part $A \cdot 8$ MeV (up to 2400 MeV)

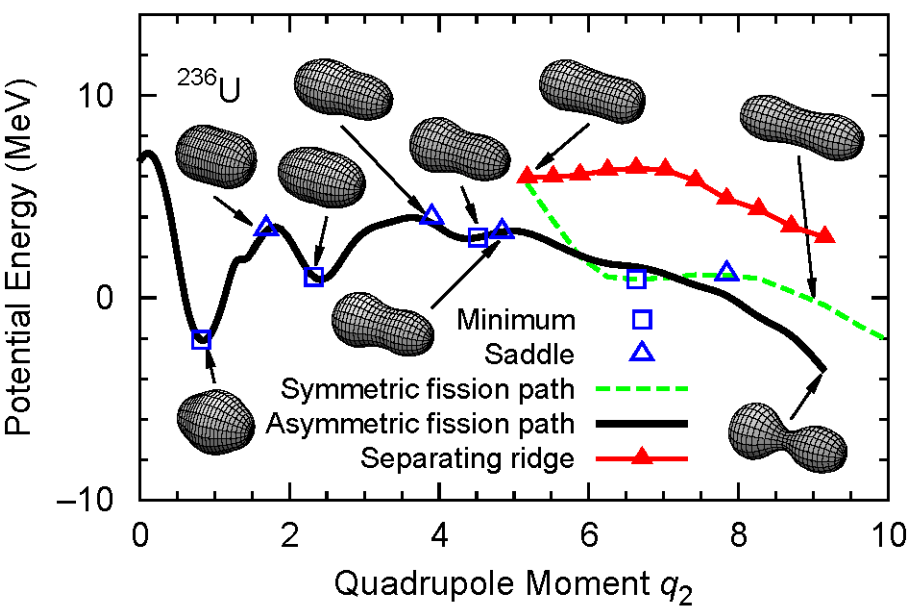
Bengtsson, Möller, Nature **449**(2007)41

- Shell effects exist not only in ground state but also at fission barrier
- Only the energy difference matters for stability of super heavies
- Calculate fission barriers !

Microscopic-Macroscopic Energy



- Asymmetric fission due to shell effects
- Energy landscape alone tells that high saddles at mass symmetry prevent asymmetric fission
- Diffusion and dynamics needed for mass distribution



Microscopic Correlation Energies

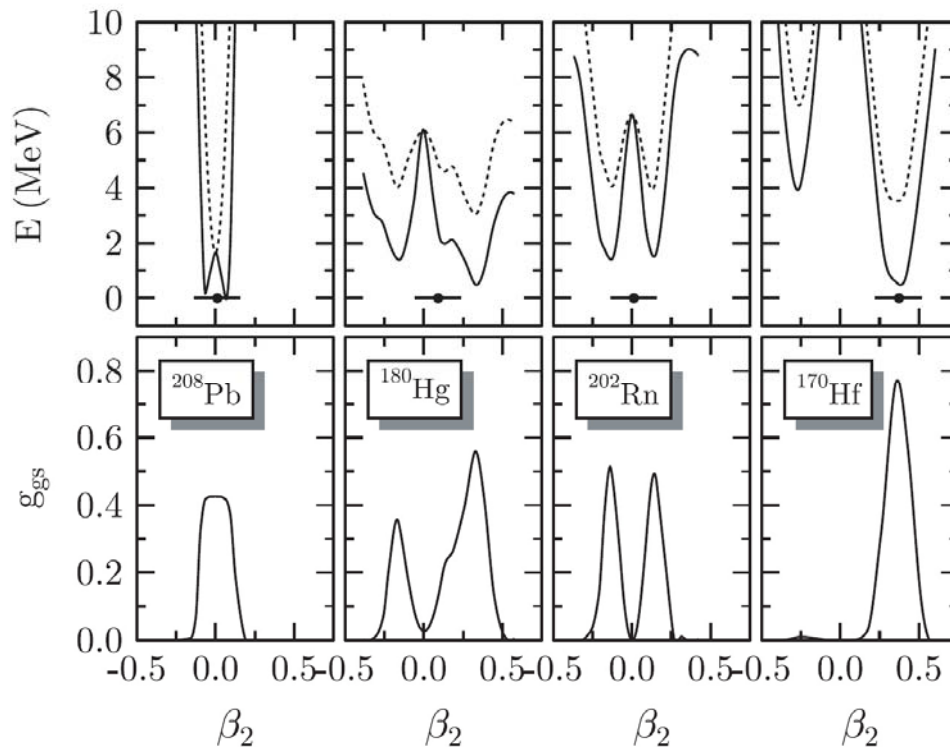


FIG. 3. Upper panel: Topography of unprojected/projected energy landscapes for typical heavy nuclei. The dotted curve denotes the energy after projection on the particle number only; the solid curve denotes the energy after projection on both particle number and angular momentum $J = 0$. The filled circle denotes the energy of the $J = 0$ projected GCM ground state. Lower panel: Collective $J = 0$ ground-state wave function. All curves and markers are drawn versus the average axial quadrupole deformation of the mean-field states they are constructed from.

Ground state:

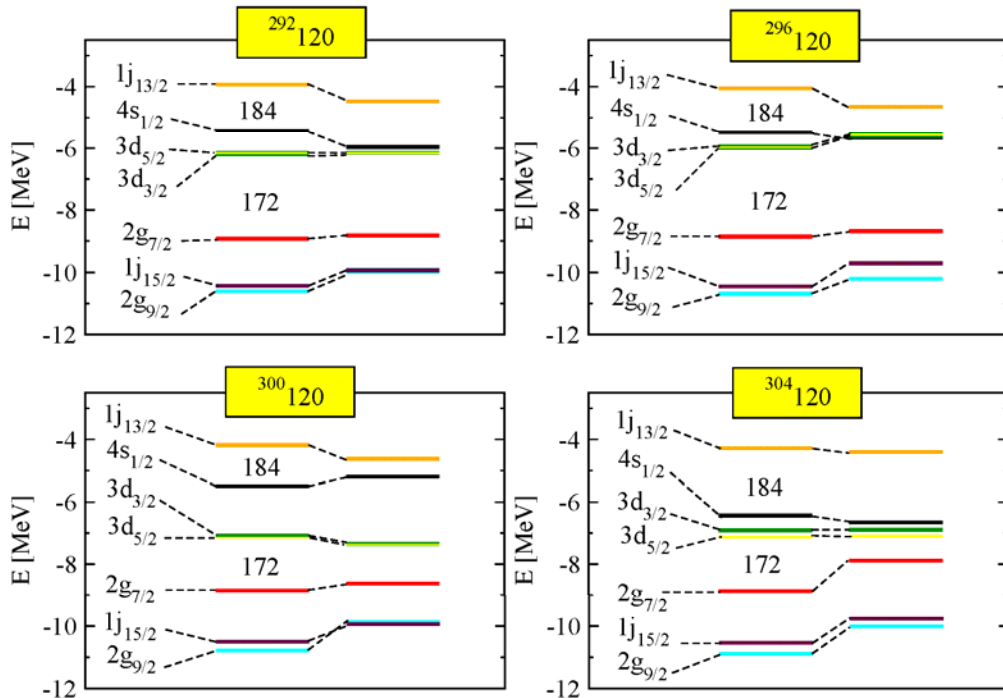
- $\Delta E \approx -2$ MeV due to project. on $J=0$
- $\Delta E \approx -0.1 \dots -2$ MeV due to mixing of states with different quadrupole mom.

Scission saddle

- What are the those numbers at fission saddle ?
Here ΔE not necessarily negative
- The difference matters for stability

Shell evolution in superheavy $Z = 120$ isotopes: Quasiparticle-vibration coupling (QVC) in a relativistic framework

1. Relativistic Mean Field + Pairing (MF): spherical minima
2. coexistence of pairing and (sub)shell closure
3. $A=300$, neutrons no clear shell gap in MF (left)
4. With additional coupling to vibrational modes (right) larger gap (~ 100 phonons below 15 MeV)

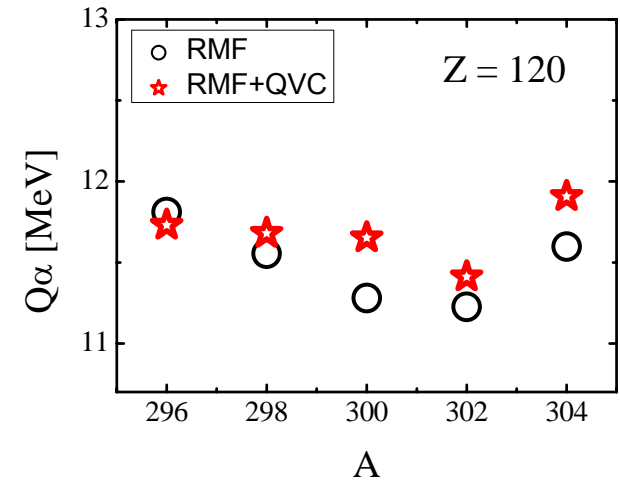


Shell stabilization & vibration (?)

Vibrational corrections to binding energy

$$E_{VC} = - \sum_{\mu} \Omega_{\mu} \sum_{k_1 k_2} |Y_{k_1 k_2}^{\mu}|^2$$

seen in α -decay Q-values



Vibrational corrections:

1. Impact on the shell gaps
2. Smearing out the shell effects

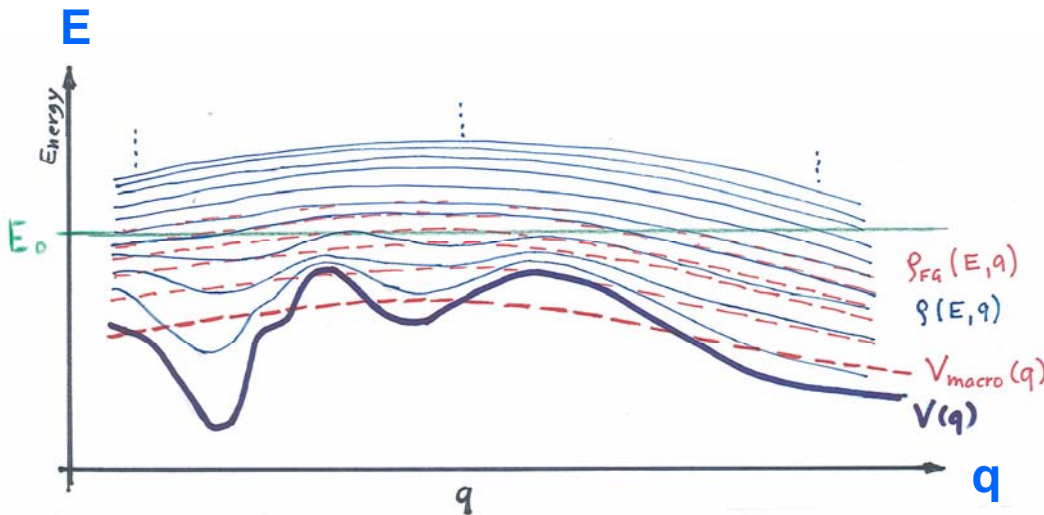
E.Litvinova, PRC 85, 021303(R) (2012)

What is the situation at fission saddle ?

Diffusive Limit

Diffusive limit:

- When collective kinetic energy negligible, system is diffusing in an energy landscape (random walk)
- Valid approximation from compound nucleus up across saddle and down to close to scission, only when neck snaps collective kinetic energy is picked up again



Input to diffusion model:

- $V(q)$ with shell and pairing effects
- Level densities $\rho(E, q)$ with shell and pairing effects
- Transition rates $q \rightarrow q+dq$
- Consistently from unified microscopic model

Summary and Outlook

Minimum request (without dynamics):

Microscopic models should look at **ground state** and **fission barrier** to guide experiment, $V(q)$

Dynamics:

Entrance to fusion and fission: Large scale nuclear motion with fluctuations

- $V(q)$ adiabatic energy landscape
- $M_{ij}(q)$ collective inertia (mass)
- $X_i(q,p) = \sum_j \gamma_{ij}(q) p_j + \delta X_i(t)$ dissipation, fluctuation

Get macroscopic transport properties from one microscopic picture, consistent and unified

Simplification: Diffusive motion (e.g. escape from compound nucleus down to scission):

- $V(q)$
- Level density
- Transition rates $q \longrightarrow q+dq$

Get all from one microscopic picture in a consistent and unified way